Smart micro‐grids
Properties, trends and local control of energy sources

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Outline

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2. The potential revolution of the smart micro-grid
3. Smart micro-grid architecture
4. The role of energy storage
5. Control issues in smart micro-grids
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9. On-line Identification of micro-grid parameters
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Smart micro-grids
Properties, trends and local control of energy sources

1. From the traditional grid to the smart grid
1. From the traditional grid to the smart grid

**The traditional grid**

- Few large power plants feeding large number of end-users
- Power plants located in strategic sites (cost-effective generation, safety)
- Centralized control (dispatcher)
- Unidirectional power flow
- Independent operation of each apparatus (the power grid performs nearly as an ideal voltage source with small internal impedance)
- No customers’ participation to power balance
1. From the traditional grid to the smart grid

**The smart grid**

- Local-scale power grids **which can operate in utility-connected or islanded mode**
- Distributed Energy Resources (DER)
- Bidirectional power flow
- Weak grid, causing interaction of power sources and loads
- Multilateral contribution to power balance
- Intelligent and controllable electronic interfaces **between energy sources and grid**
1. From the traditional grid to the smart grid

**Benefits of the smart grid**

- Distributed renewable resources
  - less carbon footprint
  - energy cost reduction
- **Energy efficiency**
  - power sources close to loads
  - improved demand response
- **Improved utilization of conventional power sources**
  - less active, reactive, unbalance and distortion power flowing through the distribution lines
- **Voltage support**
  - distributed injection of active and reactive power
- **Increased hosting capacity**
  - without investments in the grid infrastructure
1. From the traditional grid to the smart grid

**Challenges of the smart grid**

- **Bidirectional power flow**
  - need for new control and protection strategies
  - conventional voltage stabilization techniques not applicable

- **Weak grid** (non-negligible internal impedance, especially in islanded operation)
  - voltage distortion due to nonlinear loads
  - voltage asymmetry due to unbalanced loads and single-phase DER units (PV, batteries, ...)

- **Irregular power injection** by renewable energy sources
  - need for power flow regularization and peak power shaving
  - need for energy storage devices
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2. The potential revolution of the smart micro-grid
2. The potential revolution of the smart micro-grid

**LV residential micro-grids**

**General Definition:** A Smart Grid is an electrical power delivery system where power quality, efficiency and energy cost are *optimized* by pervasive use of *information and communication technology* with the aim to control *distributed energy resources*.

**Low-voltage Microgrid** = distribution system connecting a MV/LV substation with loads & distributed energy resources (DERs).

- DERs interface with the distribution grid by electronic power processors (EPP, inverters) equipped with local measurement, control and communication (*EG = Energy Gateway*).
- EGs may implement *bidirectional power control and communicate* with other generators and loads of the micro-grid to implement cooperative operation.
2. The potential revolution of the smart micro-grid

Expected benefits of micro-grids

Environment & savings
- Green power
- Full utilization of distributed energy resources
- Reduced distribution loss
- Increased hosting capacity
- Increased power quality even in remote locations
- Layered grid architecture

Social & economics
- Strengthen consumers role
- Develop communities of prosumers
- New functions and players in the energy market
- New arena for entrepreneurs, manufacturers and service providers
- New jobs for green collars

Paradigm: The INTERNET of ENERGY
2. The potential revolution of the smart micro-grid

Technological challenges

- Exploit every available energy source
- Minimize distribution losses and non-renewable energy consumption
- Increase power quality and hosting capacity
- Implement cheap ICT architectures for distributed control and communication
- Integrate micro-grid control and domotics
- Revise accounting principles and methodologies
- Restructure network protection
- Assure data security and privacy
- Pursue flexibility and scalability (from buildings to townships)
2. The potential revolution of the smart micro-grid

The future of micro-grids

- **UE Roadmap for micro-grids** (CIGRE 2010)

- **Smart grid investment forecast** (JRC report 2011)
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3. Smart micro-grid architecture
3. Micro-grid architecture

General sketch of a micro-grid
3. Micro-grid architecture

Definitions and requirements

- **Active grid nodes** correspond to *prosumers*, i.e., buildings or residential settlements equipped with distributed energy resources (DERs) and Energy Gateways (EGs)
  - **DERs** may be PV panels, wind turbines, fuel cells, batteries, flywheels, etc.
  - **EGs** include an *electronic power processor (EPP)*, capable to control the active and reactive power flow from local sources into the grid, a *local control unit (LCU)* and a *smart meter (SM)*, which provides measurement, communication and synchronization capability.
- **Passive grid nodes** correspond to traditional consumers and are assumed to be equipped with smart meters too
- **Plug & play operation** of EGs ensures *flexibility and scalability* of the micro-grid architecture
- **Distributed control and communication** allows *cooperative operation* of EGs and *synergistic utilization* of DERs
3. Micro-grid architecture

Energy Gateway – functional diagram

- Battery pack
- PV system
- Home Appliances
- Single phase residential settlement

Local (generation/storage side) $v(t), i(t)$

Local (Grid side) $v(t), i(t)$

Sync

Local control of power converters

Micro grid distributed control

Micro grid supervision/policies

Metering

Synchronization

On board ICT

Power Flows

Real Time needs

Control signals from μGrid

$P – Q$

from other gateways

$μP$

Communication

SIGNALS

μGRID

Power Electronics

DC/DC

Inverter

Metering/Billing

PLC?
3. Micro-grid architecture

Utility Interface – functional diagram

- **Energy Storage**
- **Power Electronics**
  - Aggregator, enabling distributed EGs to contribute to system-level energy management
  - Energy backup in case of islanded operation or grid dynamics
  - Micro-grid interface to the utility, managing system-level load balancing, harmonic and reactive compensation, aggregate demand response, etc.
- **I/O INTERFACE**

Concept idea: micro-grid to appear as an ‘ideal’ programmable load

- Three phase distribution infrastructure
- Communication with μ-Grid and utility
- μ GRID
- Energy gateways
3. Micro-grid architecture

Retrofitting existing plants
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4. The role of energy storage
4. The role of energy storage

**Energy efficiency – local features**

Residential settlement with loads, PV generation and energy storage connected to the mains via cabled distribution line.
4. The role of energy storage

**Power profiles**

Loads and PV generation

![Graph showing loads and PV generation over time](image)

- **P**\text{load}:
- **P**\text{PV}:

---

**Diagram:**

- 6kVA POWER CONVERTER
- 4kW PV System
- 15kWh Battery
- PCC
- \(P, Q_{\text{GEN}}\)
- \(P, Q_{\text{LOADS}}\)
- \(P_{\text{PV}}\)
- \(P_{\text{BATT}}\)
4. The role of energy storage

**Power absorption without battery**

Active power absorbed from the mains assuming

\[ P_{\text{BATT}} = 0: P_{\text{ABS}} = P_{\text{LOADS}} - P_{\text{PV}} \]
4. The role of energy storage

Distribution loss without battery

Distribution loss without battery

- **Instantaneous**
- **mean**

![Graph showing distribution loss without battery over time](image)
4. The role of energy storage

**Power absorption with battery**

\[ P_{\text{ABS}} = P_{\text{LOADS}} - P_{\text{PV}} - P_{\text{BATT}} \]

Local control tends to enforce \( P_{\text{ABS}} = P_{\text{ABS}_{\text{AVG}}} \) (daily average power)
4. The role of energy storage

Daily power profile of battery

- Power from the battery
- Discharge
- Charge

6kVA POWER CONVERTER

15kWh Battery

$P_{\text{BATT}}$
4. The role of energy storage

Daily energy profile of battery

Energy in the battery

Charge complete

Discharge limit

Charge

Discharge

kWh

time [h]

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
4. The role of energy storage

**Distribution loss without battery**

- **Distribution loss reduced by 85% (active power control only)**
4. The role of energy storage

Distributed energy storage

**Local functions** (Energy Gateways)
- Regularization of power absorption
- Reduction of losses in the distribution feeder
- Peak power shaving
- Emergency supply in case of mains outage (UPS operation)
- Node voltage stabilization
- **Prosumer energy bill reduction**

**Micro-grid functions** (Utility Interface + Energy Gateways)
- Energy sharing & backup in case of islanded operation
- Smoothing of irregular power generation by renewable sources
- Programmable active and reactive power absorption
- Power delivery to the utility on demand
- **Cost-effective energy management and ROI planning**
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5. Control issues in smart micro-grids
Local control functions (Energy Gateways)
• Exploitation of renewable energy sources (active power control)
• Management of energy storage (active power control)
• Voltage support (active & reactive power control)
• Reactive & harmonic compensation
• Load shedding & shifting

Micro-grid control functions (Utility Interface + Energy Gateways)
• Synergistic utilization of micro-grid resources
• Aggregate demand response and peak power shaving
• Load balancing by reactive current control (Steinmetz approach)
• Management of mains outages & grid dynamics
• Management of islanded operation
• Management of active and reactive power requests by the utility
5. Control issues in smart micro-grids

Hierarchic control architecture

- **Tertiary control**
  - Market operation, storage management, load scheduling
  - Aggregated demands and power flows
  - Active power dispatching (generation and demand)

- **Secondary control**
  - Ancillary services, voltage support, losses minimization, reactive power compensation
  - Voltage and current phasors $i(t), v(t)$
  - Complex power references $s(t) = p(t) + jq(t)$

- **Primary control**
  - Voltage stability, load shedding, islanding detection, inverter control
  - Instantaneous voltages $v(t)$
  - Instantaneous currents $i(t)$

- **Energy Market**
- **Distributed Control** (cooperation rules of distributed units)

- **Local control of power converters and generators**

- **Power distribution network**
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6. Inverter modeling and control
Inverter control modes

Single-phase voltage-fed full-bridge grid-connected inverters can be driven according to different control approaches:

1. **Current-mode control:** the ac-side inductor current is controlled to track a current reference set by the DC link voltage controller (typical configuration of PV systems) or by an external power loop. The inverter appears as a **Controlled current (or Power) source**.

2. **Voltage-mode control:** the current loop is driven by an external voltage control loop that tracks a voltage reference (UPS applications or droop-controlled inverters, where the power flows are controlled by acting on module and phase of the inverter ac voltage). The inverter appears as a **Controlled voltage source**.
6. Inverter modeling and control

**Inverter control: current-mode**

![Diagram of Current-Controlled / Power-Controlled Inverter]

- Energy Source & Dc-out converter stage
- PWM and Gate drivers
- Outer DC-link voltage loop or Power loop
- Inner current loop
- Current reference

**Current-controlled / Power-Controlled Inverter**
6. Inverter modeling and control

Inverter control: voltage-mode

Voltage-Controlled Inverter

Energy Source & Dc-out converter stage

PWM and Gate drivers

Inner current loop

Output Voltage reference (from droop control, P-Q control, minimum-loss control etc.)

Grid Voltage Loop

Current reference
For current-controlled inverters the usual requirement in grid-connected operation (e.g., for PV inverters) is to supply purely active power, i.e., to inject a current in phase with the line voltage ($\cos \varphi = 1$);

Assuming sinusoidal grid voltage $V_{\text{GRID}}$ and inverter current $I_{\text{GRID}}$, the phasorial representation of this operating condition is:

In general, however, the inverter can feed a current which can be leading or lagging the grid voltage:

$$\varphi = \varphi_v - \varphi_i$$
6. Inverter modeling and control

Inverter control: power injection

Complex Power supplied by the inverter:

\[ \dot{S} = V_{GRID} I_{GRID}^* = P + jQ \]

\[ \dot{S} = V_{GRID} \left( I_{GRID} e^{-j\phi} \right)^* = \]

\[ = V_{GRID} I_{GRID} e^{j\phi} = \]

\[ = V_{GRID} I_{GRID} \cos \phi + V_{GRID} I_{GRID} \sin \phi \]

\[ P = V_{GRID} I_{GRID} \cos \phi \]

\[ Q = V_{GRID} I_{GRID} \sin \phi \]

Four quadrant operation

- **P<0:** active power absorbed from the grid
- **Q>0:** inductive power injected into the grid (lagging current, \( \phi < 0 \))
- **P>0:** active power injected into the grid
- **Q>0:** inductive power injected into the grid (lagging current, \( \phi < 0 \))
- **P<0:** active power absorbed from the grid
- **Q<0:** capacitive power injected into the grid (leading current, \( \phi > 0 \))
- **Q<0:** capacitive power injected into the grid (leading current, \( \phi > 0 \))
In distributed generation, the inverters operate in the I and IV quadrants, injecting positive active power and either positive or negative reactive power.

**POWER RATING:**
The complex power that can be injected by an inverter is limited by the current and voltage rating of the components (V and I limits for the switches, I limits for the output inductors, V limit for the capacitors etc).

For a given grid voltage, these limits are represented by the apparent power

\[ A = V_{\text{GRID}} \cdot I_{\text{GRID max}} = |\dot{S}_{\text{max}}| \] [VA]
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7. Micro-grid modeling and distribution loss analysis
7. Micro-grid modeling and distribution loss analysis

**Micro-grid modeling (1)**

To analyze the micro-grid operation, a suitable **modelling** is required.

**Power Systems approach:** Network elements are represented as constant power loads / distributed generators, constant current loads, constant impedance loads. The grid is analyzed in terms of **Power Flow** relations, resulting in **nonlinear equation systems** which require numeric solvers (Newton-Raphson, etc).

**LV distribution systems:** the voltage is impressed at the Point of Common Coupling with the mains and its variation along the LV grid is within ±5% of rated value. Thus, under steady-state conditions, the constant-power loads can be represented as constant-current (or constant-impedance) elements. Similarly for the energy sources. Thus, the system model becomes linear and can be solved analytically by Kirchhoff’s and load equations.

Moreover, LV distribution lines are usually made by **cables with constant section**, i.e. impedances with constant phase (modelled as R-L series). This further simplifies the analysis, making possible the analytical solution of radial and meshed grids as well.
**Assumption**: the PCC voltage is taken as phase reference for the phasorial representation

\[
\hat{V}_{PCC} = U_{\text{rated}} + j0 = 230 + j0 \, V
\]

**Approximation**: based on the assumption of negligible phase voltage differences between grid nodes, the active and reactive currents absorbed by the loads or injected by the generators nearly coincide with the real and imaginary components of such node currents referred to the PCC voltage.

- The **real part** of the node currents controls the active power absorbed/injected at the grid nodes
- The **imaginary part** of the node currents controls the reactive power absorbed/injected at the grid nodes

\[
\Delta V << V_{PCC}
\]
7. Micro-grid modeling and distribution loss analysis

**Incidence matrix** of radial micro-grids (1)

Consider a **radial micro-grid** with \( N+1 \) **nodes** (0…\( N \)) and \( N \) **branches** (1…\( N \)), where the loads and the distributed generators are connected to the grid nodes.

The **complete incidence matrix** \( A_c \) is defined as a \( N \times (N+1) \) integer matrix whose elements are:

\[
A_c(\ell, n) = \begin{cases} 
-1 & \text{if branch } \ell \text{ leaves node } n \\
+1 & \text{if branch } \ell \text{ enters node } n \\
0 & \text{otherwise}
\end{cases}
\]

**Complete Incidence Matrix** \( A_c \)

\[
A_c = \begin{pmatrix}
-1 & 1 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 1
\end{pmatrix}
\]
The reduced **incidence matrix** $A$ is defined as a $N \times N$ integer matrix obtained by eliminating the column of node $0$ *(slack node, i.e., the Point of Common Coupling with the utility, PCC)*.

**Reduced Incidence Matrix $A$**

<table>
<thead>
<tr>
<th>Nodes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>−1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>−1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>−1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note:** The reduced Incidence Matrix $A$ is square and invertible.
7. Micro-grid modeling and distribution loss analysis

Path matrix of radial micro-grids

The transpose inverse of reduced incidence matrix $\underline{A}$ is a $N \times N$ integer matrix called path matrix $\underline{P}$, whose $n^{th}$ column gives the path from node 0 to node $n$.
7. Micro-grid modeling and distribution loss analysis

**Kirchhoff’s laws** of radial micro-grids (1)

Let $\mathbf{u}_c$ be the node voltages (including node 0) and $\mathbf{v}$ the branch voltages, the **Kirchoff’s Law for voltages** (KLV) applied to voltage phasors gives:

$$\mathbf{v} = -A_c \mathbf{u}_c \quad \Rightarrow$$

$$\begin{bmatrix}
  \dot{V}_1 \\
  \dot{V}_2 \\
  \dot{V}_3 \\
  \dot{V}_4 \\
\end{bmatrix} =
\begin{bmatrix}
  a_0 & A \\
\end{bmatrix}
\begin{bmatrix}
  \dot{U}_0 \\
  \dot{U}_1 \\
  \dot{U}_2 \\
  \dot{U}_3 \\
  \dot{U}_4 \\
\end{bmatrix}$$

In a simplified form, let $\mathbf{u}$ be the node voltages (excluding node 0), the Kirchoff’s Law for voltages (KLV) becomes:

$$\dot{\mathbf{v}} = -a_0 \cdot \dot{\mathbf{U}}_0 - A \times \dot{\mathbf{U}}$$

where $a_0$ is the first column of complete incidence matrix $A_c$. 
7. Micro-grid modeling and distribution loss analysis

**Kirchhoff’s laws** of radial micro-grids (2)

Let \( i_c \) be the node currents (including node 0, with positive polarity if leaving the grid) and \( j \) be the branch currents, the **Kirchoff’s Law for currents** (KLC) applied to current phasors gives:

\[
i_c = A^T_c j \quad \Rightarrow \begin{bmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \end{bmatrix}
\]

In a simplified form, let \( \tilde{i} \) be the node currents (excluding node 0), the Kirchoff’s Law for currents (KLC) becomes:

\[
\begin{cases} i_0 = a_0^T \times \tilde{j} \\ \tilde{i} = A^T \times \tilde{j} \end{cases}
\]
Note that: \[ i_0 = - \sum_{n=1}^{N} i_n \quad \Rightarrow \quad \dot{i}_0 = - \dot{1}_N^T \times \dot{i} \]

Thus:

\[
\dot{i}_0 = - \dot{1}_N^T \times \dot{i} = a_0^T \times \dot{j} = a_0^T \times (A^T)^{-1} \dot{i} \quad \Rightarrow \quad a_0^T \times (A^T)^{-1} = a_0^T \times \dot{P} = - \dot{1}_N^T
\]

In a simplified form, let \( \dot{i} \) be the node currents (excluding node 0), the Kirchoff’s Law for currents (KLC) becomes:

\[
\begin{align*}
\dot{i}_0 &= a_0^T \times \dot{j} \\
\dot{i} &= A^T \times \dot{j}
\end{align*}
\]
For each branch of the distribution grid we can write: \[ \dot{V}_{ij} = \dot{U}_i - \dot{U}_j = \dot{Z}_{ij} \dot{J}_{i \rightarrow j} \]

Let \( \mathbf{Z} \) be the diagonal matrix of the branch impedances, in vector form we get:

\[
\dot{\mathbf{Z}} = \text{diag}\{\dot{Z}_\ell\}_{\ell=1}^N = \begin{bmatrix}
\dot{Z}_1 & 0 & 0 & 0 \\
0 & \dot{Z}_2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \dot{Z}_N
\end{bmatrix}
\]

\[ \Rightarrow \quad \dot{\mathbf{V}} = \dot{\mathbf{Z}} \times \mathbf{J} \]
Recalling the previous definitions and results we get:

\[ \hat{V} = \hat{Z} \times \hat{J} \quad \Rightarrow \quad \begin{cases} \hat{V} = -a_0 \hat{U}_0 - A \times \hat{U} \\ \hat{I} = A^T \times \hat{J} \end{cases} \quad \Rightarrow \quad a_0 \hat{U}_0 + A \times \hat{U} = -\hat{Z} \times \left( A^T \right)^{-1} \times \hat{I} \]

The grid equations can therefore be expressed as a function of node currents and voltages in the form:

\[ A^{-1} \times a_0 \hat{U}_0 + \hat{U} = -A^{-1} \times \hat{Z} \times \hat{P} \times \hat{I} \quad \Rightarrow \quad \hat{U} = \hat{U}_0 \left( I_N^T \right) - \hat{P}^T \times \hat{Z} \times \hat{P} \times \hat{I} = \hat{U}_0 - \hat{Z}_{\text{grid}} \times \hat{I} \]
Equations $\dot{\mathbf{U}} = \dot{\mathbf{U}}_0 - \dot{\mathbf{Z}}_{\text{grid}} \times \dot{\mathbf{I}}$ represent the Thevenin-equivalent for the grid.

Equations $\dot{\mathbf{I}} = \dot{\mathbf{Z}}^{-1}_{\text{grid}} \times (\dot{\mathbf{U}}_0 - \dot{\mathbf{U}}) = \dot{\mathbf{I}}_{sc} - \dot{\mathbf{Y}}_{\text{grid}} \times \dot{\mathbf{U}}$ represent the Norton-equivalent.

The impedance matrix $\dot{\mathbf{Z}}_{\text{grid}} = \mathbf{P}^T \times \dot{\mathbf{Z}} \times \mathbf{P}$ is symmetrical.

It can be shown that the element $(m,n)$ of $\mathbf{Z}_{\text{grid}}$ represents the common impedance of the paths connecting node 0 with nodes $m$ and $n$, respectively.
7. Micro-grid modeling and distribution loss analysis

**Distribution loss in radial micro-grids**

The distribution loss is defined as:

$$P_d = \sum_{\ell=1}^{N} R_{\ell} J_{\ell_{rms}}^2 = J^T \times R \times J^*$$

- \( R \) = diagonal matrix of branch resistances

Branch currents \( J \) can be expressed as a function of node currents \( I \) as:

$$i = A^T \ j \ \iff \ j = (A^T)^{-1} \times i = P \times i$$

Thus:

$$P_d = i^T \times P^T \times R \times P \times i^* = i^T \times R_{\text{grid}} \times i^*$$

Note: \( R_{\text{grid}} \) is the real part of \( Z_{\text{grid}} \). In fact:

$$\dot{Z}_{\text{grid}} = P^T \times \dot{Z} \times P = P^T \times (R + jX) \times P = R_{\text{grid}} + jX_{\text{grid}}$$
7. Micro-grid modeling and distribution loss analysis

**Example (1)**

Simple microgrid, with 2 generators and 3 loads

Reduced incidence matrix

\[ A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \]

Remember that the slack node \( 0 \), i.e. the PCC, is not considered in the matrix

<table>
<thead>
<tr>
<th>Branch impedances</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_1 )</td>
<td>( 1+j1 , \Omega )</td>
</tr>
<tr>
<td>( Z_2 )</td>
<td>( 2+j2 , \Omega )</td>
</tr>
<tr>
<td>( Z_3 )</td>
<td>( 3+j3 , \Omega )</td>
</tr>
<tr>
<td>( Z_4 )</td>
<td>( 4+j4 , \Omega )</td>
</tr>
</tbody>
</table>

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<tr>
<th>Load currents</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{Lp1} )</td>
<td>( 5+j5 , \text{A} )</td>
</tr>
<tr>
<td>( I_{Lp2} )</td>
<td>( 10+j10 , \text{A} )</td>
</tr>
<tr>
<td>( I_{Lp4} )</td>
<td>( 15+j15 , \text{A} )</td>
</tr>
</tbody>
</table>
7. Micro-grid modeling and distribution loss analysis

Example (2)

\[ R_{\text{grid}} = A^{-1} \times R \times (A^{-1})^T = P^T \times R \times P \]

\[ A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} = P^T \]

\[ R_{\text{grid}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \text{Ω} \]

\[ R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \text{Ω} \]

<table>
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<tr>
<td>\text{Z}_1 \text{Ω}</td>
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<tr>
<td>\text{I}_{\text{Lp1}} \text{A}</td>
<td>5+j5 \text{A}</td>
</tr>
<tr>
<td>\text{I}_{\text{La2}} \text{A}</td>
<td>10+j10 \text{A}</td>
</tr>
<tr>
<td>\text{I}_{\text{Lp4}} \text{A}</td>
<td>15+j15 \text{A}</td>
</tr>
</tbody>
</table>
7. Micro-grid modeling and distribution loss analysis

Example (3)

\[ R_{\text{grid}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \Omega \]

\[ P_d = I^T \times R_{\text{grid}} \times I^* \]

\[ P_d = \begin{bmatrix} 5 + j5 & 10 + j10 & 0 & 15 + j15 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \begin{bmatrix} 5 - j5 \\ 10 - j10 \\ 0 \\ 15 - j15 \end{bmatrix} = 5350 \text{ W} \]
The analysis proposed for radial micro-grids can be applied to meshed micro-grids too, with a slightly different formulation.

In particular, the reduced incidence matrix is split in two sub-matrices: the tree sub-matrix $A_t$ and the co-tree sub-matrix $A_\ell$.

A *tree* is a generic subset of the micro-grid branches which connects all nodes and has a radial structure; the *co-tree* is the complementary subset of the micro-grid. The tree branches are called *twigs*, the co-tree branches are called *links*.

$$A = \begin{bmatrix} A_t \\ A_\ell \end{bmatrix}$$

Tree sub-matrix (includes all rows corresponding to twigs)

Co-tree sub-matrix (includes all rows corresponding to links)

The total distribution loss can be split in two terms, corresponding respectively to the twigs (tree) and the links (co-tree) giving:

$$P_d = j_t^T \times R \times j_t^* = \begin{vmatrix} j_t^T \\ j_\ell^T \end{vmatrix} \begin{vmatrix} R_t & 0 \\ 0 & R_\ell \end{vmatrix} \begin{vmatrix} j_t^* \\ j_\ell^* \end{vmatrix} = j_t^T \times R_t \times j_t^* + j_\ell^T \times R_\ell \times j_\ell^*$$
In general, the circuit theory shows that the twig currents are depended variables, which can be expressed as a function of the node currents (absorbed by the loads or injected by the generators, which are independent variables) and the link currents (flowing in the co-tree, which are independent variables too).

Application of the superposition principle gives:

\[
\begin{align*}
\mathbf{j}_t^n &= (A_t^{-1})^T \mathbf{i} = P_t \mathbf{i} \\
\mathbf{j}_t^\ell &= (A_t^T)^{-1} A_t^T \mathbf{j}_\ell = P_\ell \mathbf{j}_\ell \\
\mathbf{j}_t &= \mathbf{j}_t^n + \mathbf{j}_t^\ell = P_t \mathbf{i} + P_\ell \mathbf{j}_\ell
\end{align*}
\]

Twig currents due to node currents
Twig currents due to link currents
Total twig currents
7. Micro-grid modeling and distribution loss analysis

**Loss analysis in meshed \( \mu G \)** (3)

Correspondingly, the distribution loss can be rewritten as:

\[
P_d = \mathbf{j}_t^T \mathbf{R}_t \mathbf{j}_t^* + \mathbf{j}_\ell^T \mathbf{R}_\ell \mathbf{j}_\ell^* = \left( \mathbf{i}_t^T \mathbf{P}_t^T + \mathbf{j}_\ell^T \mathbf{P}_\ell^T \right) \mathbf{R}_t \left( \mathbf{P}_t \mathbf{i}_t^* + \mathbf{P}_\ell \mathbf{j}_\ell^* \right) + \mathbf{j}_\ell^T \mathbf{R}_\ell \mathbf{j}_\ell^* =
\]

\[
= \mathbf{i}_t^T \left( \mathbf{P}_t^T \mathbf{R}_t \mathbf{P}_t \right) \mathbf{i}_t^* + \mathbf{j}_\ell^T \left( \mathbf{P}_\ell^T \mathbf{R}_\ell \mathbf{P}_\ell \right) \mathbf{i}_t^* + \mathbf{j}_\ell^T \left( \mathbf{P}_\ell^T \mathbf{R}_\ell \mathbf{P}_\ell \right) \mathbf{i}_t^* + \mathbf{j}_\ell^T \left( \mathbf{P}_\ell^T \mathbf{R}_\ell \mathbf{P}_\ell + \mathbf{R}_\ell \right) \mathbf{j}_\ell^*
\]

Since \( \Omega_t^\ell = \left( \Omega_t^\ell \right)^T \), we can express the equation in the more synthetic form:

\[
P_d = \mathbf{i}_t^T \Omega_t^\ell \mathbf{i}_t^* + 2 \mathbf{j}_\ell^T \Omega_\ell^\ell \mathbf{i}_t^* + \mathbf{j}_\ell^T \left( \Omega_\ell^\ell + \mathbf{R}_\ell \right) \mathbf{j}_\ell^*
\]

The distribution loss depends therefore on both **node currents** and **link currents** (twig currents have been removed from the equation).

In practice, also the link currents can be expressed as a function of the node currents, which distribute among twigs and links depending on their branch impedances.
7. Micro-grid modeling and distribution loss analysis

Loss analysis in meshed μG (4)

To eliminate the dependence on the link currents it can be observed that, if all distribution cables have the same section (R/X constant), the node currents distribute among links and twigs depending on the branch resistances in a way that necessarily minimizes the distribution losses:

\[
\frac{\partial P_d}{\partial j_\ell} = 0 \quad \Rightarrow \quad 2\Omega_\ell^T i^* + 2\left(\Omega_\ell^T + R_\ell^T\right)j_\ell^* = 0 \quad \Rightarrow \quad j_\ell^* = -\left(\Omega_\ell^T + R_\ell^T\right)^{-1} \Omega_\ell^T i^*
\]

\[
P_d = i^T \Omega_\ell^T i^* + 2j_\ell^T \Omega_\ell^T i^* + j_\ell^T \left(\Omega_\ell^T + R_\ell^T\right)j_\ell^*
\]

\[
P_d = i^T \Omega_\ell^T i^* - i^T \Omega_\ell^T \left[\left(\Omega_\ell^T + R_\ell^T\right)^{-1}\right]^T \Omega_\ell^T i^* = i^T R^\text{mesh}_{\text{grid}} i^*
\]

This latter expression is formally equivalent to that applicable for radial micro-grids.
Smart micro-grids
Properties, trends and local control of energy sources

8. Optimum control of smart micro-grids
In the basic optimization process, the **distribution loss in the micro-grid** is taken as the quantity to be minimized (cost function). The motivations are:

- This is an optimum choice in terms of **energy efficiency**
- The **power consumption** of the micro-grid is minimized
- The **currents flowing in the distribution grid are minimized**; this implies that:
  - The **loads are fed by the nearest sources**, which corresponds to the most effective load power sharing among distributed generators
- The voltage drops across the branch impedances are minimized, resulting in a **voltage stabilization** effect at all nodes of the micro-grid

The optimization will be firstly done in the assumption that a **central controller** drives all energy gateways of the micro-grid and has a **complete knowledge of grid topology and impedances**

The results of such optimization are unrealistic, since several other aspects (mentioned later) should be considered. However, this sets a **benchmark** to compare the performances of any other control technique.
8. Optimum control of smart micro-grids

Distribution Loss Minimization (1)

Ideal optimization:
• Linear micro-grid modelling
• Grid topology (matrix $A$) and path impedances (matrix $Z$) known to the controller
• Unconstrained active and reactive current injection by distributed EPPs

Let
\[ i_a = K_a i \]
\[ i_p = K_p i \]
currents injected at the $N_a$ active nodes (energy gateways)
currents absorbed at the $N_p$ passive nodes (loads)

Distribution loss:
\[
P_d = i^T R_{grid} i^{*} \rightarrow P_d = i^T R_{a,a} i^{*} - 2R \left( i^T R_{a,p} i^{*} \right) + i^T R_{p,p} i^{*}
\]

where:
\[
R_{a,a} = K_a R_{grid} K_a^T, \quad R_{a,p} = K_a R_{grid} K_p^T, \quad R_{p,a} = K_p R_{grid} K_a^T, \quad R_{p,p} = K_p R_{grid} K_p^T
\]
\[
R_{a,p} = R_{p,a}^T
\]

Optimization goal
Find active node currents $I_a$ that minimize $P_d$ for a given set of load currents $I_p$
6. Optimum control of smart micro-grids

Distribution Loss Minimization (2)

Let: \( \dot{I}_a = x + jy \) \( \dot{I}_p = a + jb \)

\[
\frac{\partial P_d}{\partial \dot{I}_a} = 0 \quad \Rightarrow \quad \frac{\partial P_d}{\partial x} = 0 \quad \Rightarrow \quad 2R_{a,a} x - 2R_{a,p} a = 0 \\
\frac{\partial P_d}{\partial y} = 0 \quad \Rightarrow \quad 2R_{a,a} y - 2R_{a,p} b = 0
\]

\[\Rightarrow \quad R_{a,a} \dot{I}_a - R_{a,p} \dot{I}_p = 0\]

Observe that:

- A **centralized controller** which knows topology and impedances of the micro-grid, given the load currents, can directly drive the active nodes currents (both active and reactive terms) so as to target the minimum distribution loss condition.
- The distribution loss minimization can be done separately for the real (active) and imaginary (reactive) part of the injected currents. This may be **important in those cases when only reactive currents can be used for distribution loss minimization**, the active currents being constrained by power or energy limitations of the distributed energy resources (renewable sources, batteries, etc.).
8. Optimum control of smart micro-grids

Application example (1)

Simple microgrid, with 2 generators and 3 loads

**STEP 1:** Reduced incidence matrix

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Branches</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = \begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ -1 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \ -1 &amp; 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; -1 &amp; 1 &amp; 0 \end{bmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Branch impedances</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
</tr>
<tr>
<td>$Z_2$</td>
</tr>
<tr>
<td>$Z_3$</td>
</tr>
<tr>
<td>$Z_4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load currents</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{Lp1}$</td>
</tr>
<tr>
<td>$I_{Lp4}$</td>
</tr>
<tr>
<td>$I_{Lp4}$</td>
</tr>
<tr>
<td>$I_{La2}$</td>
</tr>
</tbody>
</table>

Load currents

$\begin{bmatrix} 5+j5 \text{ A} \\ 10+j10 \text{ A} \\ 15+j15 \text{ A} \end{bmatrix}$
8. Optimum control of smart micro-grids

Application example (2)

STEP 2: Inverse of incidence matrix

\[
A^{-1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1
\end{bmatrix}
\]

STEP 3: Matrix of branch resistances:

\[
\Omega = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 4
\end{bmatrix}
\]

STEP 4: Matrix 

\[
R_{\text{grid}} = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 3 & 1 & 1 \\
1 & 1 & 4 & 4 \\
1 & 1 & 4 & 8
\end{bmatrix}
\]

\[
R = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 4
\end{bmatrix}
\]

\[
\Omega
\]

\[
\Omega
\]
Given the above matrices, the inherent distribution loss (with all inverters switched off) can be derived as a function of load currents $I_L$:

$$P_{do} = \tilde{I}_L^T R_{grid} \tilde{I}_L^* = [5 + j5 \quad 10 + j10 \quad 0 \quad 15 + j15] \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \begin{bmatrix} 5 - j5 \\ 10 - j10 \\ 0 \\ 15 - j15 \end{bmatrix} = 5350 \text{ W}$$

**STEP 5:** Matrices $K_a$ and $K_p$ (identify active and passive nodes)

$$K_a = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad K_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**STEP 6:** Sub-matrices of $R_{grid}$

$$R_{a,a} = K_a R_{grid} K_a^T, \quad R_{a,p} = K_a R_{grid} K_p^T$$
$$R_{p,a} = K_p R_{grid} K_a^T, \quad R_{p,p} = K_p R_{grid} K_p^T$$

$$R_{a,a} = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}, \quad R_{a,p} = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$
$$R_{p,a} = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}, \quad R_{p,p} = \begin{bmatrix} 1 & 1 \\ 1 & 8 \end{bmatrix}$$
STEP 7: Calculation of the optimum currents to be injected at the active nodes given the currents $I_p$ absorbed by the loads at the passive grid nodes. Let:

The optimum currents are:

$$i_{a, opt} = R_{a,a}^{-1} R_{a,p} i_p = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 5 + j5 \\ 15 + j15 \end{bmatrix} = \begin{bmatrix} -1.3636 - j1.3626 \\ -15.9091 - j15.9091 \end{bmatrix}$$

In practice, currents $I_a$ can be expressed as the difference between the currents absorbed by the loads connected at the active grid nodes ($I_{La}$) and the currents injected by the distributed energy resources at the same nodes ($I_{Ga}$). Thus:

$$\bar{i}_a = \begin{bmatrix} \bar{i}_{La_2} - \bar{i}_{Ga_2} \\ \bar{i}_{La_3} - \bar{i}_{Ga_3} \end{bmatrix} \Rightarrow \bar{i}_{Ga_2} = \begin{bmatrix} 10 + j10 \\ 0 \end{bmatrix} - \begin{bmatrix} -1.3636 - j1.3626 \\ -15.9091 - j15.9091 \end{bmatrix} = \begin{bmatrix} 11.3636 + j11.3626 \\ 15.9091 + j15.9091 \end{bmatrix}$$

STEP 8: Calculation of the distribution loss in the optimum condition

$$P_{d, opt} = \bar{i}_a^T R_{a,a} \bar{i}_a^* + 2 \Re \left( \bar{i}_a^T R_{a,p} \bar{i}_p^* \right) + \bar{i}_p^T R_{p,p} \bar{i}_p^* = 1827.3W$$
8. Optimum control of smart micro-grids

Remarks

The above loss minimization approach represents a first step towards optimum control. In practice, the optimization procedure can be extended to consider also:

- **islanded operation**, when the micro-grid is disconnected by the utility \( i_0 = 0 \)
- **inverter losses**, which affect the distribution efficiency since the inverters manage the full power generated by the distributed energy resources
- **current capability of the inverters**, which actually limits the active and reactive power deliverable at the active grid nodes
- **actual power capability of distributed generators and energy capability of distributed energy sources**, which constraint the active power deliverable at the active grid nodes
- **other aspects**, like *intermittent power generation* of renewable sources, *lifetime optimization of storage batteries*, *daily cost of energy* and *revenues from power trading* that might influence the optimization process in a wider perspective, both technical and economic
Smart micro‐grids
Properties, trends and local control of energy sources

9. On-line Identification of micro-grid parameters
The previous optimization has been done in the assumption that a central controller has a complete knowledge of grid topology and impedances. In this section we analyze some techniques which allow on-line evaluation of node-to-node distances and identification of micro-grid topology. These techniques take advantage of the capabilities of modern powerline communication (PLC) technologies, which are particularly suited for micro-grid applications.

In fact, in low-voltage residential micro-grids, the same power lines connecting the users can be used to convey data. The small distances between users and the absence of transformers make possible a direct powerline communication among grid nodes, without requiring any additional communication infrastructure. The on-line identification approach can also be extended to estimate the line impedances.

However, in a residential micro-grid the size of the distribution cables is usually the same, thus the knowledge of node-to-node distances is sufficient to run the optimum control algorithm, as well as the distributed quasi-optimum control techniques which will be discussed in the next sections.
Node-to-node communication

- Node-to-node communication architecture
- Node-to-node distance measurement

Standard PRIME
(PoweRline Intelligent Metering Evolution)

**PRIME overview:**
- Designed for outdoor applications
- OFDM physical layer
- Maximum bit rate 128kbps
- Transmission over CENELEC A band, in the range 45kHz-92kHz with 97 equally spaced sub-carriers
- MAC layer needs to be “customized” to fit peer-to-peer communication (PRIME is originally master-slave)

Node-to-node distance measurement: PLC enables the use of TOA (Time Of Arrival) techniques, currently under testing over ≈1km of real distribution cables in the Smart Micro-Grid Facility at DEI
The knowledge of grid map (incidence matrix), node-to-node distances and branch impedances is generally required to implement loss minimization techniques. In practice, the knowledge of branch impedances is not required if the distribution lines have constant section. In this case, node-to-node distances are sufficient.

**PLC-based distance measurement by Time of Arrival (TOA) ranging technique**

- Node A broadcasts a data packet, which is received by node B at time $\tau_B$
- Node B waits a fixed time $T$ and then replies to A with another data packet.
- A receives the packet at time $\tau_A = 2\tau_B + T$
- Time $\tau_B$ depends on the distance $d_{AB}$ between nodes A and B by the relation $\tau_B = d_{AB}/c$, with $c$ the speed of light.

Thus:

$$d_{AB} = \frac{c(\tau_A - T)}{2}$$

Distance measurement accuracy 1.5-10 m
Grid mapping algorithm

- If the ranging procedure is repeated for each pair of nodes in the micro-grid, the *distance matrix* $D$ can be determined, whose generic element $d_{mn}$ gives the distance between nodes $m$ and $n$.

- We say that **two nodes $n$ and $m$ are neighbors** if their distance is the minimum among the lengths of all paths connecting them, i.e.:

\[ d_{nm} < d_{nk} + d_{km}, \quad k = 1 \ldots N \]

- Neighbor nodes are directly connected by a branch of the distribution grid, thus each pair of neighbor nodes identifies a row of the complete incidence matrix $A_c$.

- The reduced incidence matrix $A$ is then obtained by suppressing the column corresponding to node 0 (slack node).

- Finally, the tree and co-tree sub-matrices $A_t$ and $A_l$ are derived by partitioning $A$ into a full-rank (tree) sub-matrix and the residual (co-tree) sub-matrix.
Smart micro-grids
Properties, trends and local control of energy sources

10. Distributed surround control of smart micro-grids
10. Distributed surround control

Introduction

• In this section we analyze a distributed plug & play control technique, called surround control, which provides local minimization of the distribution losses, resulting in a quasi-optimum operation of the entire micro-grid.

• The technique requires that every grid node, both active and passive, is equipped with a smart meter, i.e., a local measurement unit capable of data processing and powerline communication.

• This allows identification of both the incidence matrix (network topology) and the distance matrix (node-to-node distances), extended to active and passive nodes.

• Given the incidence matrix, each active node identifies the neighbor nodes, i.e., the active nodes connected by a direct link and the passive nodes fed by such links.

• Then, a local optimum control algorithm is applied, which only requires data exchange among neighbor nodes.

• The proposed control technique ensures flexibility and scalability, i.e., it can be applied irrespective of micro-grid architecture, and automatically adapts when a new node is implemented in the micro-grid.
10. Distributed surround control

Token ring sequential control

The distributed grid-connected inverters cyclically update their ac current references (control phase).

Outside the control phase, the inverters keep constant their ac current references (hold phase).

When an inverter is in the control phase, the neighbors keep the hold phase. This prevents possible detrimental control interactions.

The distributed EPPs (grid-connected inverters) operate as current sources (to stabilize the grid impedances).
10. Distributed surround control

Conduction loss in distribution lines

Let $r_{AB}$ be the resistance per unit of length of the line, the distribution loss in the line between active nodes A and B is given by:

$$P_{LOSS} = \sum_{k=0}^{K} r_{AB} \Delta_k |i_k|^2 = \sum_{k=0}^{K} r_{AB} \Delta_k i_k i_k^*$$

where:

$$i_k = i_{AB} - \sum_{\ell=1}^{k} i_{L\ell}$$

$$i_k = -\left(i_{BA} - \sum_{\ell=k+1}^{K} i_{L\ell}\right)$$

Given the currents absorbed by the passive loads fed along path A-B, the distribution loss in path A-B can therefore be expressed as a function of active node current $I_{AB}$ (or $I_{BA}$).
10. Distributed surround control

**Optimization goal:** find the values of $I_{AB}$ and $I_{BA}$ that minimize the conduction losses in path A-B

\[
\frac{\partial P_{LOSS}}{\partial i_{AB}} = 0 \quad \frac{\partial P_{LOSS}}{\partial i_{BA}} = 0
\]

\[
\begin{aligned}
I_{AB}^{opt} &= \frac{1}{d_{AB}} \sum_{k=1}^{K} i_{Lk} d_{Bk} \\
I_{BA}^{opt} &= \frac{1}{d_{AB}} \sum_{k=1}^{K} i_{Lk} d_{Ak}
\end{aligned}
\]

The optimum node currents depend only on the loads and their distribution along path A-B

Moreover:

\[
\begin{aligned}
i_{AB} &= i_{AB}^{opt} \\
i_{BA} &= i_{BA}^{opt}
\end{aligned} \quad \iff \quad \hat{U}_A = \hat{U}_B
\]

**Distribution path connecting active nodes A and B**
10. Distributed surround control

Loss minimization in distribution lines (2)

In general, nodes A and B are not equipotential, thus:

\[ i_{AB} = i_{AB}^{\text{opt}} + \frac{\dot{U}_A - \dot{U}_B}{\dot{z} d_{AB}} = i_{AB}^{\text{opt}} + i_{AB}^{\text{circ}} \]

\[ i_{BA} = i_{BA}^{\text{opt}} + \frac{\dot{U}_B - \dot{U}_A}{\dot{z} d_{AB}} = i_{BA}^{\text{opt}} + i_{BA}^{\text{circ}} \]

Impedance per unit of length of distribution line

Circulation current

Optimum current
Consider a cut-set of the micro-grid

The current at node N can be expressed as:

\[ i_N = \sum_{k=1}^{K} i_{N_k} = \sum_{k=1}^{K} i_{N_k}^{\text{opt}} + \sum_{k=1}^{K} \frac{\dot{U}_N - \dot{U}_k}{Z_k} \]

\[ \dot{i}_N^{\text{circ}} = 0 \quad \Rightarrow \quad \begin{cases} 
\dot{i}_N = i_{N_k}^{\text{opt}} \\
\dot{U}_N = \dot{U}_N^{\text{opt}} = \frac{\sum_{k=1}^{K} \dot{U}_k}{\sum_{k=1}^{K} \frac{1}{Z_k}} 
\end{cases} \]

- Depends on loads connected to paths \( L_1 - L_K \)
- Depends on voltage differences

**Minimum distribution loss condition**

- EPP in control phase
- EPPs in hold phase
10. Distributed surround control

Node current/voltage optimization

**Node current optimization**

\[ i_N = i_N^{opt} = \sum_{k=1}^{K} i_{Nk}^{opt} = \sum_{k=1}^{K} \frac{1}{d_{Nk}} \sum_{m=1}^{M_{Nk}} i_{Lm}^{Nk} d_{m}^{Nk} \]

This equation holds separately for active and reactive terms, thus optimization can be done by acting on active currents, reactive currents, or both.

**Node voltage optimization**

\[ \dot{U}_N = \dot{U}_N^{opt} = \sum_{k=1}^{K} \frac{\dot{U}_k}{\dot{Z}_k} \approx \sum_{k=1}^{K} \frac{\dot{U}_k}{d_k} \]

This method is very sensitive to voltage measurement errors.

The computation of optimum node current (EPP reference current) requires distance estimation (ranging), local grid mapping, and current measurement at surrounding passive nodes.

The computation of optimum node voltage (EPP reference voltage) requires local grid mapping, knowledge of path impedances (or node-to-node distances), and voltage measurement at surrounding active nodes.
10. Distributed surround control

**Node current/voltage optimization**

**Node current optimization**

\[
i_N = i_N^{\text{opt}} = \sum_{k=1}^{K} i_{Nk}^{\text{opt}} = \sum_{k=1}^{K} \frac{1}{d_{Nk}} \sum_{m=1}^{M_{Nk}} i_{Nk}^{Lm} d_m^{Nk}
\]

Optimum current control does not excite network dynamics!

In fact injecting currents at the grid nodes affects marginally the node voltages, thus grid operation is not influenced.

**Node voltage optimization**

\[
\dot{U}_N = \dot{U}_N^{\text{opt}} = \sum_{k=1}^{K} \frac{\dot{U}_k}{\dot{Z}_k} \approx \sum_{k=1}^{K} \frac{\dot{U}_k}{d_k}
\]

Optimum voltage control does excite network dynamics!

In fact, changing the voltage at node N may cause significant variations of the line currents, thus affecting also the voltages of the other nodes in the micro-grid.
10. Distributed surround control

**Current/voltage relation at node N**

1. Given the optimum node voltage and current, assuming the same impedance $z$ per unit of length for all distribution paths, from the measured voltage and current at node N we estimate this impedance as:

   \[
   \dot{z} = \frac{\dot{U}_N - \dot{U}_N^{opt}}{\dot{i}_N - \dot{i}_N^{opt}} \sum_{k=1}^{K} \frac{1}{d_{Nk}}
   \]

2. The Thevenin equivalent circuit at node N (characterized by internal impedance and no-load voltage) can be determined as:

   \[
   \dot{Z}_N^{eq} = \dot{z} \left( \sum_{k=1}^{K} \frac{1}{d_{Nk}} \right)^{-1}
   \]

   \[
   \dot{U}_N^o = \dot{U}_N^{opt} - \dot{Z}_N^{eq} \dot{i}_N^{opt}
   \]

3. The general relation between voltage and current at node N is expressed by:

   \[
   \dot{U}_N = \dot{U}_N^o + \dot{Z}_N^{eq} \dot{i}_N
   \]

   **This latter equation allows conversion of voltage references into current references and vice versa (current-mode $\leftrightarrow$ voltage-mode control)**
Token ring control

- A token moves along the micro-grid, and only the active node \((N)\) keeping the token is enabled to modify its current reference according to the minimum distribution loss criterion.

- When an active node receives the token, it:
  1. collects voltage phasors from neighbour nodes
  2. measures (or recalls) the distances from neighbour nodes
  3. computes the optimum voltage reference
  4. computes the current reference variation needed to reach the optimum voltage
  5. sends the token to the next active node

\[
\dot{U}^\text{opt}_N \approx \sum_{k=1}^{K} \frac{\dot{U}_k}{d_{nk}} / \sum_{k=1}^{K} \frac{1}{d_{nk}}
\]

\[
\Delta i_N = \frac{\dot{U}^\text{opt}_N - \dot{U}_N}{\dot{Z}^\text{eq}_N}
\]
Convergence of control algorithm

- **Note:** The minimum loss condition is reached when all nodes are equipotential, their voltage being equal to the voltage impressed at the PCC.

- The control theory shows that this condition is progressively approached if the nodes which sequentially receive the token drive their voltage toward the value:

  \[
  \dot{U}_{\text{ref}}^N = \sum_{k \in [1,K]} b_{Nk} \dot{U}_k
  \]

- The choice of coefficients \(b_{Nk}\) defines how fast the algorithm converges to the steady state optimum condition.

- The convergence condition for the control algorithm is:

  \[
  \sum_{k \in [1,K]} b_{kN} = 1, b_{kN} \geq 0
  \]

- This condition is satisfied with surround control since we assume:

  \[
  b_{Nk} = \frac{1}{d_{Nk}} \left/ \sum_{k=1}^{K} \frac{1}{d_{Nk}} \right.
  \]
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11. Distributed cooperative control of smart micro-grids
Surround Control ensures minimum distribution loss, but requires a full knowledge of the micro-grid topology, which requires data exchange among all grid nodes. Moreover, it has strict requirements in terms of node-to-node communication and synchronization (PMU, Phasor Measurement Unit), which are not easily satisfied with cheap commercial technology.

**Question:** there is a different distributed control technique which has easier implementation and still keeps good performances? *(Sub-optimum solution)*

**Remark:** Beyond the mathematical analysis, an intuitive interpretation of distribution loss minimization is that “the distribution loss reduces if the loads are supplied by the generators nearby”
11. Distributed cooperative control

Principle of cooperative control

Cooperative control approach

1. Each load $m$ splits its active and reactive power demand $P_m$ and $Q_m$ among the active nodes $n$ in inverse proportion to their distances:

$$P^m_n = \frac{P_m}{d^m_n} \left( \sum_{n=0}^{N} \frac{1}{d^n_m} \right)^{-1} = P_m \frac{d^n_{eq}}{d^n_m} \quad \Rightarrow \quad \sum_{n=1}^{N} P^m_n = P_m$$

$$Q^m_n = \frac{Q_m}{d^m_n} \left( \sum_{n=0}^{N} \frac{1}{d^n_m} \right)^{-1} = Q_m \frac{d^n_{eq}}{d^n_m} \quad \Rightarrow \quad \sum_{n=1}^{N} Q^m_n = Q_m$$

2. Each active node $n$, within its current capability, supplies the total power requested by the passive loads:

$$P_n = \sum_{m=1}^{M} P^m_n = \sum_{m=1}^{M} P_m \frac{d^n_{eq}}{d^n_m} \quad Q_n = \sum_{m=1}^{M} Q^m_n = \sum_{m=1}^{M} Q_m \frac{d^n_{eq}}{d^n_m}$$
Advantages of cooperative control

- Use of PMUs (phasor measurement units) can be avoided, since the loads address their requests in terms of active and reactive power, which are conservative quantities and do not depend on the phase of the node voltages.
- There is no need for micro-grid topology identification, since only the node-to-node distances are requested to implement the control algorithm.

Disadvantage of cooperative control

- The solution can diverge from the optimum condition in case of saturation of the current capability of the inverters.

Upgrade of cooperative control

- The saturation conditions must be properly managed by shifting the power requests from the saturated active nodes to the non-saturated nodes.
Managing saturation

The splitting algorithm of the load power is modified as follows:

\[
P^n_m = P_m \frac{\beta_{nP}^n}{d_m^n} \left/ \sum_{n=0}^{N} \beta_{nP}^n \frac{d_m^n}{d_m^n} \right. \\
Q^n_m = Q_m \frac{\beta_{nQ}^n}{d_m^n} \left/ \sum_{n=0}^{N} \beta_{nQ}^n \frac{d_m^n}{d_m^n} \right.
\]

where:

\[
\beta_{nP}(k) = \beta_{nP}(k-1) \cdot \alpha_{nP}(k) \quad \beta_{nP_{min}} \leq \beta_{nP}(k) \leq 1 \\
\beta_{nQ}(k) = \beta_{nQ}(k-1) \cdot \alpha_{nQ}(k) \quad \beta_{nQ_{min}} \leq \beta_{nP}(k) \leq 1 \\
\beta = \text{sampling interval (sampling frequency = 10 Hz)}
\]

- Coefficients \( \alpha \) express the residual power capability of active nodes (\( \alpha < 1 \) means saturated current capability).
- Coefficients \( \beta \) represent the corrective terms applied to the ideal power distribution criterion (inverse of distance). \( \beta < 1 \) means limited contribution due to saturation, \( \beta = 1 \) means full contribution.
- At every sampling interval coefficients \( \beta \) are updated: they can be further reduced if saturation still holds, while can be increased (up to 1) if saturation disappears (e.g., due to a reduction of load power request).
11. Distributed cooperative control

Upgraded cooperative control (2)

Managing saturation
The splitting algorithm of the load power is modified as follows:

\[
P^n_m = P^n_m \frac{\beta^n_{nP}}{d^n_m} / \sum_{n=0}^{N} \beta^n_{nP} \quad Q^n_m = Q^n_m \frac{\beta^n_{nQ}}{d^n_m} / \sum_{n=0}^{N} \beta^n_{nQ}
\]

where:

\[
\beta^n_{nP}(k) = \beta^n_{nP}(k-1) \cdot \alpha^n_{nP}(k) \quad \beta^n_{nP_{min}} \leq \beta^n_{nP}(k) \leq 1
\]

\[
\beta^n_{nQ}(k) = \beta^n_{nQ}(k-1) \cdot \alpha^n_{nQ}(k) \quad \beta^n_{nQ_{min}} \leq \beta^n_{nP}(k) \leq 1
\]

\[
\alpha^n_{nP}(k) = \frac{P^n_{max}}{P^n(k-1)}
\]

\[
\alpha^n_{nQ}(k) = \frac{Q^n_{max}}{Q^n(k-1)}
\]

Advantages

- The power limits of the active nodes are automatically met
- Recovery from saturation happens quickly
- Load power requests are met precisely
- The power splitting criterion approaches the “minimum distance” criterion as close as possible, within the power limits of the active nodes
- Control is inherently stable
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12. Simulation results
12. Simulation results

Simulation approach

Assumptions

- The proposed control techniques have been validated by simulation in the Matlab – Simulink environment.

- To minimize the complexity of simulation and to reduce the simulation times a phasorial simulation tool has been developed.

- The graphs showing the time behaviour of the system represent must be interpreted as sequences of steady states (quasi-stationary behavior), where fast dynamics are neglected.
12. Simulation results

**Simulation Example (1)**

### 18-bus LV network

<table>
<thead>
<tr>
<th>DG</th>
<th>$P_{\text{MAX}}$ kW</th>
<th>$S_{\text{MAX}}$ kVA</th>
<th>Load $Z = R + j\omega L$</th>
<th>Power @ 230V RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>2</td>
<td>L1 5 kW cos(\phi) = 0.91</td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>1</td>
<td>2</td>
<td>L2 5 kW cos(\phi) = 0.91</td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td>3</td>
<td>5</td>
<td>L3 2.5 kW cos(\phi) = 0.96</td>
<td></td>
</tr>
<tr>
<td>G4</td>
<td>3</td>
<td>5</td>
<td>L4 2.5 kW cos(\phi) = 0.96</td>
<td></td>
</tr>
<tr>
<td>G5</td>
<td>3</td>
<td>5</td>
<td>L5 2.5 kW cos(\phi) = 0.96</td>
<td></td>
</tr>
<tr>
<td>G6</td>
<td>1</td>
<td>2</td>
<td>L6 5 kW cos(\phi) = 0.91</td>
<td></td>
</tr>
<tr>
<td>G7</td>
<td>10</td>
<td>15</td>
<td>L7 10 kW cos(\phi) = 0.80</td>
<td></td>
</tr>
<tr>
<td>G8</td>
<td>10</td>
<td>15</td>
<td>L8 10 kW cos(\phi) = 0.80</td>
<td></td>
</tr>
<tr>
<td>G9</td>
<td>10</td>
<td>15</td>
<td>L9 10 kW cos(\phi) = 0.80</td>
<td></td>
</tr>
</tbody>
</table>

**Total Loads**

- Total DERs / EPPs

**Total length of distribution line** 1.8 km
12. Simulation results

Simulation Example (2)

Initial situation: Inverters OFF

Loss: \( P_{R_{\text{max}}} = 1.2 \text{ kW} \)

Voltages:
Generators nodes voltages - RMS value

The distributed control techniques are analyzed in specific operating conditions, their performance being compared with those of optimum control.

Two cases are considered:

- **Active and Reactive** current control constrained only by converters saturation (to show the achievable performances in a real system with power generation & energy storage)
  - In practice, actual active power capability is determined by energy storage & generated power constraints (sun, wind, batteries, etc), while reactive power can be regulated within the current capability of the inverters.

- **Purely reactive current control** constrained by converters saturation (to show the achievable improvement without power generation & energy storage)

The actual micro-grid performances are intermediate between these two cases.
12. Simulation results

**Surround control** (1)

---

**Surround control neglecting saturation**

**Active and reactive current control**

A Token Ring approach is adopted, where the 9 generators are activated and updated in sequence. Every 9 token jumps, the loads update their current demands.

---

**After 100 iterations all loads reduce their power absorption to 50% of the nominal ratings**

---

**Total loss**

- Actual loss
- Theoretical minimum (1)
- Theoretical minimum (2)

---

**Injected active and reactive RMS currents**

---
12. Simulation results

**Surround control (2)**

**Surround control considering saturation**

**Active and reactive current control**

A Token Ring approach is adopted, where the 9 generators are activated and updated in sequence. Every 9 token jumps, the loads update their current demands.

---

**After 100 iterations all loads reduce their power absorption to 50% of the nominal ratings**
12. Simulation results

**Cooperative Control (1)**

Cooperative control without saturation management

Active and reactive current control

A Token Ring approach is adopted, where the 9 generators are activated and updated in sequence. Every 9 token jumps, the loads update their current demands.

After 100 iterations all loads reduce their power absorption to 50% of the nominal ratings
Cooperative control with Saturation Management

Active and reactive current control

12. Simulation results

Cooperative control (2)
Assuming that only reactive currents are injected in the grid by the distributed grid-connected inverters, the distribution losses become:

- **Surround Control** \( P_{\text{LOSS}} = 928\text{W} \) (23% loss reduction)
- **Cooperative Control** \( P_{\text{LOSS}} = 935\text{W} \) (22% loss reduction)

This represents the worst case condition, i.e., the case of a micro-grid without energy storage capability and distributed power generation.

- The presence of distributed power generators allows a first level of improvement, since their active power can partially compensate for the active power demand of local loads.
- The situation is further improved if the grid-connected inverters can manage the energy of storage devices too, because this allows a local compensation for the entire active and reactive power demand by the loads, resulting in minimum distribution losses.
12. Simulation results

Purely reactive current control

[Graph showing total loss with current loss, theoretical minimum (1), and theoretical minimum (2)]

Cooperative control with Saturation Management (pure reactive current control)

After 100 iterations all loads reduce their power absorption to 50% of the nominal ratings
Conclusions

1. Smart micro-grids represent a fast-growing and challenging arena for ICT, power electronics and power systems research and applications.

2. The bottom-up revolution made possible by an extensive implementation of the micro-grid paradigm can have a dramatic impact on the entire value chain of the electrical market.

3. A structured multi-layer reorganization of the electrical grid can provide huge benefits in terms of energy savings, quality of service and flexibility of operation, without altering the physical infrastructure of the grid.

4. The development of suitable distributed control & communication techniques can provide flexibility, scalability, power quality, integration and exploitation of any kind of energy resources, energy efficiency and stability of operation.

5. The successful Internet paradigm can possibly be replicated in the domain of distributed energy generation, distribution and utilization.
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Research activities at DEI/UniPD

DEI team

DEI smart grid facility